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## ABSTRACT

To determine whether the range of ability within the classroom has a significant effect on students' performance in mathematics, and to identify the class compositional or instructional characteristics that contribute to differences between homogeneous and heterogeneous classes in terms of mathematics achievement, an analysis was conducted using data from the Second International Mathematics Study. The sample consisted of 1,319 eighth graders in 79 classes from 61 schools. Data included pretest and posttest scores on the study's mathematics test, student and family characteristics, and student attitudes toward mathematics concerning effort. Hierarchical linear modeling (HLM) was used to study relationships. Results support previous research that found no advantage to being in a class with either heterogeneous or homogeneous ability levels. There was a non-significant effect of range of ability on the outcome. Results also point to the differentiating effect of previous achievement on subsequent mathematics achievement. Advantages of the HLM procedure are discussed. Four tables present study findings. An appendix describes the student-level variables. (SLD)

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## Hierarchical Linear Modeling of Class Ability Range on Student Mathematics Achievement

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Paper presented at a poster session at the annual meeting  
of the American Educational Research Association  
Atlanta, GA April 12-16, 1993

## Hierarchical Linear Modeling of Class Ability Range on Student Mathematics Achievement

Substantial research has been conducted on the effects of student ability grouping, both across classes and within classrooms. Many issues are involved in the decision to restrict the range of ability across and within classrooms but essentially the desire to ease the teaching burden of adapting materials and teaching groups of students with content and pace appropriate to their ability level is at the basis of such decisions. Grouping for instruction, however, has the effect of decreasing the amount of time available for instruction and restricting the content presented to each instructional group. Sorensen & Hallinan (1977), for example, found that students who were ability-grouped for instruction were exposed to less of the curriculum but learned more of the portion of the curriculum to which they were exposed.

In many studies, the issue of homogeneity of class ability is confounded by comparing the results of the instruction and subsequent learning of high-ability versus low-ability students. According to Barr & Dreben (1983), while gearing the curriculum to student ability is considered a beneficial effect and is viewed favorably in the research, tracking is considered deleterious to some students (low-ability students) and is generally viewed unfavorably. But both are different ways of looking at the same practice and are aspects that have to be reconciled regardless of approach taken.

There are benefits and unintended consequences of both approaches in addition to whether or not it makes a difference which approach is used in terms of student achievement. The simple question of whether or not the range of class ability makes a difference has not been addressed recently, especially when using an analytic technique which accounts for differences within and across classes.

According to Burstein (1980), investigations of educational effects are inherently multilevel and using a single unit of analysis procedure does not deal appropriately with the multilevel nature of the educational data. One problem with using student-level data for all variables is that educational treatments are not administered independently to individuals; individuals within classes have shared experiences. This dependence of observations cannot be ignored. One

problem with aggregating data to the class or school level is that it typically inflates the estimated effects of background on outcomes and decreases the likelihood of identifying effective teacher-classroom-school characteristics.

As with most research on school effects, previous research on the effects of range of ability within a class on student achievement has used a single level of analysis--either student-level or class-level. The results of previous research have shown an inconsistent results. Some studies (Edmiston & Benefer, 1949; Justman, 1968; and Good & Marshall, 1984) found that students in heterogeneous classes performance better; others (Rothrock, 1961; Bergun, Swanson, & Sawyer, 1966; and Slavin & Karweit, 1985) found that students in homogeneous classes performed better; and still others (Miller & Otto, 1930; Koontz, 1961; and Esposito, 1973) found there was no difference between the two groups.

These results may be due to a true similarity or difference in performance of students in classes with narrow or wide ranges of ability or to the fact that the statistical techniques previously used were not able to detect differences that existed. With the introduction of multilevel modeling techniques for analyzing nested data, therefore, the previous research needs to be revisited using an appropriate statistical technique.

The objectives of this study were twofold: 1) to determine whether the range of ability within a classroom has a significant effect on students' performance in mathematics; and 2) to identify the class compositional or instructional characteristics that contribute to differences between homogeneous and heterogeneous classes in terms of student mathematics achievement.

## METHODS

### Sample

One of the requirements for using HLM in studying the effects of class-level characteristics on student performance is that data for intact (or of substantial size) classes of students within schools is available. One of the few national databases that was so sampled was the Second International Mathematics Study

(SIMS). In SIMS, schools were sampled such that they were representative of the national schools in terms of geographic area and school size, classes within schools were then randomly sampled, and students within intact classes were included in the sample, along with the teachers of those classes (Westbury, Caroll, & Thalathoti, 1989).

The SIMS study sampled students in remedial, typical, enriched, and algebra classes; only students in typical classes were included in this study. The sample in the current study consisted of 1319 U.S. eighth-grade students in 79 classes from 61 schools. Students and teachers/classes with missing data were dropped from the sample.

### Instruments

As part of SIMS, mathematics achievement tests were administered to students at the beginning and end of eighth grade and questionnaires containing demographic and attitudinal information were administered to teachers and students who participated in that study. Student data used in the present study consisted of pretest and posttest scores on the mathematics achievement test developed for the study; student and family background characteristics, such as student sex and ethnic group classification and parental education and occupations; and student attitudes toward mathematics concerning effort. Teacher and class data used in the present study consisted of: teacher education and experience, classroom composition variables such as class size and range of class ability, and instructional process variables such as percentage of time spent in small-group instruction, and coverage of tested material in the curriculum.

### Procedures

Hierarchical linear modeling (HLM) is the analytic technique used in this study. HLM is a multilevel regression program developed by Bryk, Raudenbush, Steltzer, and Congdon (1988) that uses maximum likelihood empirical Bayes techniques to converge on regression estimates whose efficiency is enhanced by the EM algorithm developed by Dempster, Laird, and Rubin (1977). HLM first models the variability in student mathematics achievement as a function of

student-level variables. Then the coefficients thus obtained are used as outcome variables to be predicted by class-level data. Using this technique, one is able to determine the relative contribution of specific between-class variables on the variability in the within-class parameters.

Procedurally, a zero or ANOVA model is defined in which the proportion of variance between and within classes is determined. A base model consisting only of student-level variables is then defined to determine the effect of the student-level variables on the outcome. Comparing the results of the base and zero model enables one to determine the proportion of variance accounted for by the addition of the student-level variables. Next, a compositional model, in which class averages of the variables used at the student-level are introduced along with the variable of interest, range of class ability, to determine whether, as previous research has indicated, compositional effects exist. Comparison of the compositional and base models enables one to determine the proportion of variance accounted for by the addition of the compositional class-level variables. Finally, an explanatory model is defined to determine which of the teacher, class, or instructional characteristics suggested by previous research contribute to differences in student achievement across classes. Comparison of the explanatory and base models enables one to determine the proportion of variance accounted for by the addition of the explanatory class-level variables.

### Analysis

In this study, previous research and ordinary least squares regression were used initially to identify significant student-level predictors of mathematics achievement. The effect of student sex was found to be nonsignificant on the outcome so that variable was dropped from the model. When majority/minority status and SES were included in the same model, the results of SES were nonsignificant, so SES was dropped in favor of majority/minority status. The final variables included in the student-level analysis consist of: student majority/minority status; previous student achievement level (pretest mathematics achievement score); and student effort (scale developed from attitudinal data collected from the students in the SIMS study).

Class-level data of interest are included in the modeling of the effect of class-level variables on student achievement across classes. Aggregates of the student-level variables that serve as compositional variables consist of the average level of student effort in the class, the average pretest score for the class, the proportion of minority students within the class, and the proportion of males in the class. Explanatory class-level variables consist of teacher characteristics (teacher experience and education), class characteristics (range of class ability and class size) and characteristics of the instructional program (use of grouping for instruction, percent of time spent in small group instruction, level of breadth or depth of curriculum coverage, and the classes' opportunity to learn the material tested). A complete listing and description of the variables used in this study is presented in the appendix.

## RESULTS

The results of the analysis of the HLM zero, base, compositional, and explanatory models are presented below. Table 1 contains the results of the zero or ANOVA model in which only the outcome score is allowed to vary. These results show that the grand mean of the mathematics achievement score is 18.421 which is significantly different from zero at an alpha level or at least .001. The class means for mathematics achievement significantly vary across classes, also at least at the .001 level.

TABLE 1

FIXED EFFECTS:

	GAMMA(*)	STANDARD ERROR	T STATISTIC	P-VALUE
FOR BASE COEF.				
BASE	18.421271	0.571526	32.232	0.000

RANDOM EFFECTS:

PARAMETER	ESTIMATED PARAMETER VARIANCE	DEGREES OF FREEDOM	CHI SQUARE	P-VALUE
BASE COEF.	22.99132	78	791.58	0.000

Table 2 contains the results of the base model in which three student-level variables are added to the modeling of mathematics achievement. These results show that controlling for student effort, majority/minority status, and previous mathematics achievement, the average score is 18.349 which is significantly different from zero at an alpha level of at least .001. Effort has a significant positive effect on the outcome (significant at an alpha level of at least .001); students with high levels of effort average 18.623 on the mathematics test and those with low levels of effort average 18.075. Majority/minority status also have a significant positive effect on the outcome (significant at an alpha level of at least .05); majority students have an average mathematics score of 19.447 and minority student have an average of 17.254. Finally, previous achievement has a significant positive effect on the outcome (significant at an alpha level of at least .001); students with higher pretest scores have average posttest scores of 19.067 and student with lower pretest scores have average posttest scores of 17.631.

The results also show that, controlling for student effort, majority/minority status, and previous achievement, the mathematics achievement outcomes still vary significantly across classes (the chi-square is significant at an alpha level of at least .001). The relationship between pretest and posttest scores also varies significantly across classes (the chi-square is significant at an alpha level of at least .01) but the relationship of effort and majority/minority status on the outcome is the same across classes (the chi-squares did not meet the criterion for significance at an alpha level of .05). Thus the variability in the outcome that remained is due to sampling variance and could not be attributed to differences in teacher, class, or instructional characteristics. In subsequent models, therefore, student effort and majority/minority status are treated as fixed variables and the outcome and previous achievement are treated as random variables. As such, class-level variables are used to model the variance across classes only in the outcome and in the relationship of previous achievement to the outcome.

TABLE 2

## FIXED EFFECTS:

		GAMMA(*)	STANDARD ERROR	T STATISTIC	p-VALUE
FOR	BASE COEF.				
	BASE	18.348980	0.578348	31.727	0.000
FOR EFFORT	SLOPE				
	BASE	0.274394	0.030214	6.997	0.000
FOR MAJMIN	SLOPE				
	BASE	1.097779	0.468541	2.343	0.019
FOR PRESCOR	SLOPE				
	BASE	0.717963	0.034671	20.708	0.000

## RANDOM EFFECTS:

PARAMETER	ESTIMATED PARAMETER		DEGREES OF FREEDOM	CHI SQUARE	P-VALUE
		VARIANCE			
BASE COEF.	24.99257	58	1104.7	0.000	
EFFORT SLOPE	0.02263	58	70.399	0.127	
MAJMIN SLOPE	2.81342	58	69.415	0.145	
PRESCOR SLOPE	0.03163	58	87.504	0.008	

NOTE: THESE VALUES ARE BASED ON ONLY 59 OF 79 UNITS THAT HAD SUFFICIENT DATA FOR COMPUTATION

Table 3 contains the results of the compositional model in which the range of student ability within the class and aggregates of the student-level variables are added to the base model. These results show that the average level of effort and previous achievement in a class has positive significant effects on the outcome (significant at least at the .001 level for previous achievement and at least at the .01 level for effort) while whether the class was homogeneous or heterogeneous in ability level and the proportion of minority students within the class has nonsignificant effects on the outcome. The average level of student effort and the range of class ability has significant positive effects on the relationship between the pretest and posttest scores (significant at least at the .005 level for effort and at least at the .05 level for range) but the effects of the proportion of minority students within the class and the average pretest level of the class are not significant.

TABLE 3

## FIXED EFFECTS:

		GAMMA(*)	STANDARD ERROR	T STATISTIC	p-VALUE
FOR	BASE COEF.				
	BASE	4.835747	1.761539	2.777	0.006
	RANGL	0.059027	0.145119	0.171	0.865
	PROPMIN	0.024308	0.0115126	1.566	0.117
	AVGPRL	1.022636	0.108161	9.461	0.000
	AVGEIT	0.646581	0.12	5.351	0.008
FOR	EFFORT SLOPE*				
	BASE	0.275708	0.0115126	7.845	0.000
FOR	MAJMIN SLOPE*				
	BASE	1.224463	0.408159	3.000	0.003
FOR	PRESCOR SLOPE				
	BASE	0.782701	0.115126	6.046	0.000
	RANGL	0.083744	0.030163	2.320	0.020
	PROPMIN	0.000009	0.001757	0.005	0.996
	AVGPRE	-0.000645	0.0115126	-0.056	0.956
	AVGEFF	0.072212	0.02116	3.015	0.003

\* - THE RESIDUAL VARIANCE FOR THIS PARAMETER HAS BEEN SET TO ZERO.

## RANDOM EFFECTS:

	ESTIMATED PARAMETER	DEGREES	OF FREEDOM	CHI SQUARE	P-VALUE
PARAMETER	VARIANCE				
BASE COEF.	6.00304	74		446.09	0.000
PRESCOR SLOPE	0.02116	73		113.25	0.006

These results also show that even after adding these compositional variables, there is still sufficient variability in the outcome and in the relationship between the pretest and posttest scores to be accounted for by other variables not included in this model.

Table 4 contains the results of the explanatory model in which characteristics of the class, teacher, and instruction are included in the modeling of the outcome and the relationship between previous and subsequent achievement. These results show that none of these characteristics has a significant effect on the outcome and that only the amount of class time spent in small group instruction is the only characteristic to have a significant effect on the relationship between previous achievement and the outcome (significant at the .005 level).

The results also show that the addition of these variables accounts for almost all of the variability in the relationship between pretest and posttest scores across classes while the amount of variability in the outcome is still statistically significant (at least at the .001 level).

TABLE 4

FIXED EFFECTS:

		GAMMA(*)	STANDARD ERROR	T STATISTIC	P-VALUE
FOR	BASE COEF.				
	BASE	6.020666	2.695495	2.234	0.035
	TCHREXP	0.094302	0.131776	0.716	0.306
	TCHRED	0.074235	0.173537	0.428	0.362
	CLASSIZE	-0.085851	0.050946	-1.685	0.097
	RANGE	-0.027383	0.362812	-0.075	0.396
	OPP2LRN	-0.000307	0.057375	-0.005	0.397
	EMPHASIS	-0.173363	0.146049	-1.187	0.196
	GRPINST	0.707964	0.799729	0.885	0.267
	SGTIME	0.002069	0.016254	0.127	0.394
	AVGPRE	1.076540	0.101048	10.654	0.000
	AVGEFF	0.794553	0.267554	2.970	0.006
FOR	EFFORT SLOPE*				
	BASE	0.271213	0.035203	7.704	0.000
FOR	MAJMIN SLOPE*				
	BASE	1.215288	0.408451	2.975	0.006
FOR	PRESGOR SLOPE				
	BASE	0.521978	0.274844	1.899	0.067
	TCHREXP	0.003522	0.012000	0.294	0.380
	TCHRED	0.002270	0.016740	0.136	0.393
	CLASSIZE	0.000339	0.004788	0.071	0.396
	RANGE	0.080101	0.036350	2.204	0.037
	OPP2LRN	0.005175	0.005687	0.910	0.261
	EMPHASIS	-0.002093	0.014233	-0.147	0.393
	GRPINST	-0.001223	0.080611	-0.015	0.397
	SGTIME	0.004553	0.001701	2.677	0.013
	AVGPRE	0.001694	0.010073	0.168	0.391
	AVGEFF	0.078054	0.025556	3.054	0.005

\* - THE RESIDUAL VARIANCE FOR THIS PARAMETER HAS BEEN SET TO ZERO.

RANDOM EFFECTS:

PARAMETER	ESTIMATED PARAMETER	DEGREES			
		VARIANCE	OF FREEDOM	CHI SQUARE	P-VALUE
BASE COEF.	6.29858	68		407.08	0.000
PRESGOR SLOPE	0.01695	78		98.181	0.061

In terms of the proportion of variance accounted for in each of these models, the initial analysis shows that 63.5% of the variance in the outcome exists within classes and that only 36.5% exists between classes; therefore, only 36.5% of the total observed variance is explainable by class-level characteristics.

A comparison of the parameter variance estimated in the zero model and in the base model shows that the addition of the student-level variables to the model accounts for 50% of the variability in the outcome. The variables included in the compositional model account for 76% of the variance in student scores across classes and 33% of the variance in the relationship between pretest and posttest performance across classes. Finally, the addition of explanatory variables accounts for no additional variance in the outcome but about 19.8% of the variability in the effect of previous achievement on the outcome.

#### DISCUSSION

The results from the analyses conducted in this study do not concur with previous research that found that there was a difference in the effects of either heterogeneity or homogeneity in class ability on student achievement. It instead confirmed previous research that concluded that there was no advantage to being in a class with either heterogeneous or homogeneous ability levels. Analysis of the data in this study shows a nonsignificant effect of range of ability on the outcome. This indicates that students in homogeneous and heterogeneous classes are likely to have about the same level of mathematics achievement.

However, the results also point to the differentiating effect of previous achievement on student's subsequent mathematics achievement. In homogeneous classes one's previous mathematics achievement level is a crucial factor in one's subsequent mathematics achievement. This result partially confirms the result obtained by Rowan & Miracle (1983) that cross-class ability grouping has a deleterious effect on low-ability students by reinforcing initial inequalities in achievement. In heterogeneous classes, however, previous achievement is not a crucial factor. One might say that homogeneity of class ability has the effect of maintaining the status quo whereas heterogeneity of class ability has the effect of breaking the hold of previous achievement level on student performance and

allowing subsequent mathematics achievement to regress toward the mean. This means, in effect, that the effect of heterogeneous classes is negative for initially higher-scoring students but is positive for initially lower-scoring students, a result that is only partially favorable.

The results also show some significant effects of compositional variables on the outcome. Students in classes with higher class averages and higher levels of effort tend to have higher mathematics achievement. However, being in a class with a higher level of effort is the only compositional variable that has an effect on the relationship of previous achievement on subsequent mathematics achievement. In addition to having heterogeneity of ability within a class, the class has to have high levels of effort in order to be able to break the hold of previous achievement on student's subsequent mathematics achievement. Possibly because of restriction of range, being in a class with higher initial achievement levels does not influence the differential effect of previous achievement on the outcome. It is probably unlikely that a class with high initial achievement could be very heterogeneous in ability.

When explanatory variables consisting of characteristics of the class, teacher, and instructional program are included in the modeling of mathematics achievement, they have no effect on the outcome. However one explanatory variable, the amount of time spent in small-group instruction, does influence the differential effect of the range of class ability on the outcome. This indicates that spending more time in small-group instruction has a positive effect on the outcome in heterogeneous classes but not in homogeneous classes. This result partially confirms the result found by Rowan & Miracle (1983) that within-class ability grouping has a positive effect on the performance of low-ability students.

In terms of accounting for difference in outcome across classes, the base model shows that the relationships between majority/minority status and effort and the outcome are the same across classes. The addition of the explanatory variables to the modeling of the effect of previous achievement on subsequent mathematics achievement accounts for all the possible variability in scores across classes. Therefore, only the modeling of the outcome still has sufficient variability left to be modeled by variables not included in this study.

The discussion now turns to methodological issues. The methodology used in this study is more appropriate for analyzing multilevel data. An important factor in using this technique for modeling the effects of student- and class-level variables on mathematics achievement is the ability to decompose the total variance into between-class and within-class components. The results of the zero or ANOVA model indicate that almost two-thirds of the variability in mathematics achievement is within-class rather than between-class; therefore, the variance accounted for by student-level variables is compared to the variance that exists within the class rather than the total variance in the outcome. Approximately half of the variance at the student level is accounted for by the student-level variables included in this model; approximately half, therefore, is accountable to variables not included in this model and would be subject to additional research.

While the student-level regression coefficients obtained using HLM are comparable to what would have been obtained had either a student-level or class-level analyses been conducted, the standard errors and the proportion of variance accounted for using student-level analysis would be underestimates of the actual amount of error because single level analyses don't take into account the dependence among responses within classrooms as HLM does. The standard errors using multilevel modeling are more similar to the standard errors obtained when class-level analysis, where standard errors are more accurately estimated, is used.

Other advantages of using HLM to model mathematics achievement is the ability to model variation for only those variables in which the outcomes vary across schools and to compare the proportion of variance accounted for by class-level variables to the amount of variance existing across classes (36.5%) instead of to the total variance in the outcome. Thus, unlike either student-level or class-level analyses, when using multilevel modeling one is able to distinguish between parameter variance and sampling variance and compare the effects of class-level variables to just that portion that represents parameter variance. The resulting proportions of variance accounted for thus becomes quite substantial; approximately three-quarters of the variability in outcome across classes and one-third of the differential effect of previous achievement on the outcome are accounted for by including class-level variables in the model.

## REFERENCES

Barr, R., & Dreeben, R. (1983) How Schools Work. Chicago: University of Chicago Press.

Bergun, M.M., Swanson, L.W., & Sawyer, D.M. (1966). An experiment on homogeneous grouping for reading in elementary classes. Journal of Educational Research, 59, 413-414.

Burstein, L. (1980). The role of level of analysis in the specification of educational effects. In R. Dreeben & J. A. Thomas (Eds.). The Analysis of Educational Productivity. Volume I: Issues in Microanalysis. Cambridge MA: Ballinger.

Bryk, A. S., Raudenbush, S. W., Seltzer, M., & Congdon, R. T. (1988). An Introduction to HLM: Computer Program and Users Guide. Version 2. Chicago: Department of Education, University of Chicago.

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm (with discussion). Journal of the Royal Statistical Society (Series B) 39: 1-8.

Edmiston, R.W., & Benefer, J.G., (1949). The relationship between group achievement and range of abilities within groups. Journal of Educational Research, 42, 547-548.

Esposito, D. (1973). Homogeneous and heterogeneous ability grouping; principal findings and implications for evaluating and designing more effective educational environments. Review of Educational Research, 43, 163-179.

Good, T.L., & Marshall, S. (1984). Do students learn more in heterogeneous or homogeneous groups? In P.P. Peterson, L.C. Wilkinson, & M.T. Hallinan (Eds.), The social context of instruction, (pp. 15-38). New York: Academic Press.

Justman, J. (1968). Reading and class homogeneity. Reading Teacher, 21, 314-316.

Koontz, W.F. (1961). A study of achievement as a function of homogeneous grouping. Journal of Experimental Education, 30, 249-253.

Miller, W.S., & Otto, H.J. (1930). Analysis of experimental studies in homogeneous grouping. Journal of Educational Research, 21, 95-102.

Rothrock, D.G. (1961). Heterogeneous, homogeneous, or individualized approach to reading. Elementary English, 38, 233-235.

Rowan, B., & Miracle, A. (1983). Systems of ability grouping and the stratification of achievement in elementary schools. Sociology of Education, 56, 133-144.

Slavin, R.E., & Karweit, N.L. (1985). Effects of whole class, ability grouped, and individualized instruction on math achievement. American Educational Research Journal, 22, 351-367.

Sorensen, A.B., & Hallinan, M.T. (1977). A reconceptualization of school effects. Sociology of Education, 50, 273-289.

Westbury, I., Caroll, C., & Thalathoti, V. (1989). Second International Mathematics Study: United States Population A - Technical Summary. International Association for the Evaluation of Educational Achievement. Urbana IL: University of Illinois at Urbana/Champaign College of Education.

## APPENDIX

### Description of Student-Level Variables

SEX	Dummy variable: "0" = females; "1" = males
SES	Three-item scale consisting of categories of mother's and father's education and father's occupation
MAJMIN	Dummy variable: "0" = minority group; "1" = majority group
EFFORT	Eight-item scale consisting of items from a student questionnaire:  My parents really want me to do well in mathematics. I feel challenged when I am given a difficult mathematics problem. No matter how hard I try I still do not do well in mathematics. * I usually understand what we are talking about in mathematics class. I will work a long time in order to understand a new idea in mathematics. I really want to do well in mathematics. I refuse to spend a lot of my own time doing mathematics. * If I had my choice I would not learn any more mathematics. *
	* negatively stated items reversed
PRESCOR	Raw score on a 40-item mathematics test administered at the beginning of eighth grade
POSTSCOR	Raw score on a 40-item mathematics test administered at the end of eighth grade

### Description of Class-Level Variables

RANGE	Collapsing of categorical variable ranging from very wide to very narrow range into effect codes with "-1" representing narrow range of ability and "1" representing wide range.
PROPMAL	Class aggregate of the SEX student-level variable; the proportion of students within the class who were male
AVGSES	Class aggregate of the SES student-level variable; the average SES level for students within the class
PROPMIN	Class aggregate of the MAJMIN student-level variable; the proportion of students within the class who were minorities
AVGEFF	Class aggregate of the EFFORT student-level variable; the class average score on the EFFORT scale
AVGPRE	Class aggregate of the PRESCOR student-level variable; the average pretest score within the class